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Guang Xiao, Nan Yang, Renyu Zhang

Olin Business School, Washington University in St. Louis, St. Louis, Missouri 63130
{xiaoguang@wustl.edu, yangn@wustl.edu, renyu.zhang@wustl.edu}

We consider a periodic review joint pricing and inventory control model in which a firm faces both stochastic demand and fluctuating procurement costs. To address procurement cost fluctuation, the firm adopts a dual-sourcing strategy, under which it procures from a spot market with immediate delivery and through a forward-buying contract with postponed delivery. Our analysis offers the unique insight that a risk-neutral firm may earn higher expected profit under a more volatile procurement cost process. This is because the firm makes its pricing and sourcing decisions in response to the realized cost in each period. Moreover, we characterize how the firm should dynamically adjust its pricing and sourcing decisions in accordance to cost evolution. For example, if sourcing through the forward-buying contract is less expensive than sourcing directly from the spot market, the optimal safety stock is decreasing in the current spot market purchasing cost. However, the optimal order quantity through the forward-buying contract is, in general, not monotone in the current spot-purchasing cost. Finally, we conduct extensive numerical experiments to show that dynamic pricing and dual sourcing may be either strategic complements or substitutes in the presence of fluctuating procurement costs and uncertain demand. This is because dynamic pricing mitigates demand uncertainty risk and exploits procurement cost fluctuation, whereas dual sourcing may either intensify or dampen demand risk.

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1. Introduction

Firms in today's unstable economic environment face growing procurement cost fluctuation because of changes in commodity prices, production costs, and global economic conditions (e.g., currency exchange rates). For example, Hewlett-Packard (HP) has long been challenged by the fluctuating procurement cost of its product components. For instance, the procurement cost of DRAM (dynamic random-access memory) dropped by more than 90% in 2001 and then more than tripled in early 2002 (Nagali et al. 2008). To tackle the combined risk from product demand uncertainty, procurement cost fluctuation, and component availability, HP has adopted both supply diversification and dynamic pricing strategies. On one hand, HP launched its procurement risk management (PRM) program in the early 2000s. The PRM program adopts the portfolio sourcing approach consisting of spot, long-term, and short-term, and other sourcing possibilities, and it gives rise to a $425 million reduction in procurement cost over a six-year period (Billington 2002, Nagali et al. 2008). On the other hand, as documented by Billington et al. (2002), HP also dynamically prices its products based on procurement cost and inventory availability. More specifically, in HP's standard portfolio management process, the firm regularly reviews the prices for all its products

and makes adjustments as appropriate (*TonerNews* 2014). For instance, in response to rising production and supply chain costs, HP adjusted the price of its LaserJet toner cartridges by 5% on September 1, 2008, affecting a total of 156 products (see Hewlett-Packard 2008). The firm also increased the list prices of 49 HP LaserJet accessories and long life consumables (LLC) SKUs by 20% on February 1, 2013. The price increases were in response to global economic conditions, including currency volatility (see Hewlett-Packard 2012).

In the face of the growing procurement cost fluctuation, many firms, such as HP, have taken both the multisourcing strategy and the dynamic pricing strategy. The two strategies are usually executed by two separate units of a firm (the procurement unit and the marketing unit). Thus, a key question faced by such firms is how these two strategies can be optimally coordinated under demand uncertainty and procurement cost fluctuation. To address this question, we propose a periodic-review single-item stochastic inventory model in which the random demand is stochastically decreasing in the sales price, and the procurement cost is driven by an exogenous underlying Markov process. The firm has two sourcing options: to procure from a spot market with immediate delivery and through the forward-buying

contract with postponed delivery (one-period lead time). This model enables us to examine the optimal pricing and sourcing decisions of a risk-neutral profit-maximizing firm in the face of stochastic demand and fluctuating procurement costs. We study the impact of procurement cost fluctuation upon the firm's profit, as well as its pricing and sourcing decisions, and the strategic relationship between dynamic pricing and dual sourcing when the firm faces uncertain demand and fluctuating procurement costs.

Our analysis shows that, under a mild condition on the cost process, a risk-neutral firm prefers a more volatile procurement cost process, because it earns higher expected profit when future procurement cost is larger in the convex stochastic order. As the firm makes the pricing and sourcing decisions in *response* to the cost realization in each period, it can effectively react to cost fluctuation by extracting the high profit with low posterior costs to compensate for the low profit with high posterior costs, thus profiting from the cost volatility. As a result, a more volatile cost process creates more opportunities than risks and generates higher expected profit.

The procurement cost volatility drives the firm to pass (part of) the cost fluctuation to its customers. More specifically, when facing a higher current procurement cost or anticipating a higher future cost trend, the firm should increase the sales price of its product. However, the impact of current procurement cost upon the optimal inventory decision of the firm is more involved, because current procurement cost contains information about future costs. With a higher procurement cost, the firm faces the trade-off between ordering less to save current procurement cost and ordering more to speculate in anticipation of higher future costs. We show that the relative cost between spot purchasing and forward buying may significantly alter the firm's response to cost changes. More specifically, if spot purchasing is more expensive, the optimal safety stock decreases in the current procurement cost. On the other hand, if forward buying is more expensive, the firm increases the safety stock in response to a higher current cost. The optimal forward-buying quantity is, in general, nonmonotone in the current procurement cost. In summary, the firm should dynamically adjust its pricing and sourcing decisions in response to stochastic cost changes. Such responsiveness, as discussed above, enables the firm to benefit from the cost volatility.

In addition, we investigate the impact of dual-sourcing flexibility (captured by the ratio between the effective cost of the forward-buying contract and the spot market purchasing cost) upon the optimal pricing and inventory policy. A lower ratio increases the value of dual sourcing, and implies higher dual-sourcing flexibility. The firm with higher dual-sourcing flexibility benefits more from the less risky forward-buying channel. As a result, it orders less from the spot market and less intensively passes the cost risk to its customers.

Our extensive numerical studies show that dynamic pricing and dual sourcing may be either strategic complements or substitutes. Dynamic pricing helps mitigate the demand uncertainty risk, whereas the additional sourcing channel in dual sourcing may either fortify or dampen the demand risk. When the firm adds the less responsive forward-buying sourcing channel via the dual-sourcing strategy, the flexibility to control demand via pricing becomes more valuable, so dynamic pricing and dual sourcing are strategic complements. On the other hand, if the more responsive spot purchasing is added into the dual-sourcing portfolio, the responsiveness of dynamic pricing is less valuable, so, in general, the two strategies are substitutes.

To conclude this section, we summarize our main contribution as follows: (1) To the best of our knowledge, we are the first to study the joint pricing and inventory management model with fluctuating procurement costs and dual sourcing. We analyze how procurement cost fluctuation influences the optimal pricing and procurement policy. (2) We deliver the unique insight that a risk-neutral firm may earn higher expected profit under a more volatile procurement cost process, because the firm makes its pricing and sourcing decisions in response to the cost realization in each period. (3) We numerically show that under uncertain demand and fluctuating procurement costs, dynamic pricing and dual sourcing may be either strategic complements or substitutes, depending on whether the additional sourcing channel in the dual-sourcing portfolio intensifies or dampens demand uncertainty risk.

2. Literature Review

This work is mainly related to two streams of literature: (1) inventory management with spot market price fluctuation and (2) joint pricing and inventory management under stochastic demand.

There is a large body of literature that studies the optimal inventory control under fluctuating spot market prices in different scenarios. The main result in this line of literature is that a procurement cost dependent base-stock policy is optimal in the inventory system without fixed ordering cost, and a cost dependent (s, S) policy is optimal with fixed ordering cost. We refer interested readers to Haksöz and Seshadri (2007) for a comprehensive review. At the embryonic era of inventory theory, Fabian et al. (1959) first study the optimal timing and purchasing quantity of the raw material when its price fluctuates. Kalymon (1971)

analyzes a dynamic stochastic inventory model with purchasing setup cost where the future prices of the item are driven by a Markov process. Gavirneni (2004) shows that a price-dependent order-up-to policy is optimal if the spot market price evolves in a Markovian fashion. Devalkar et al. (2011) study a firm that buys an input commodity, converts it into a processed commodity, and sells the processed commodity through multiperiod forward contracts. They find that the optimal procurement and processing decisions are governed by price-dependent inventory thresholds. Recently, continuous review models have also been studied in the literature. For example, Feng and Pang (2010) study the optimal coordination of production planning and sales admission when the firm sells its product via both spot market and a long-term supply channel. Yang and Xia (2009) identify conditions under which optimal decisions are monotone in spot market prices in a continuous time model. Using the single unit decomposition approach, Berling and Martínez de Albéniz (2011) characterize the optimal base-stock policy when the purchasing cost is a geometric Brownian motion or an Ornstein–Uhlenbeck process.

In the literature on inventory management with spot market price fluctuation, the multisourcing problem in the presence of both spot market and other procurement contracts has received considerable attention. Li and Kouvelis (1999) develop valuation methodologies for different types of risk-sharing supply contracts in the face of uncertain spot market prices. Flexible contract under uncertain spot market prices is also studied by Sethi et al. (2004), with focus on quantity flexibility. Li et al. (2001) study the optimal horizontal coordination between production units located in different countries with fluctuating production costs. Yi and Scheller-Wolf (2003) investigate an inventory system where the firm can procure from either a capacitated contract supplier or the spot market with setup cost. They show that an (s, S) type policy is optimal when unmet demand is backlogged. A similar dual-sourcing problem is studied by Chen et al. (2013). They analyze a stochastic inventory system, which replenishes from the spot market and a long-term contract supplier, and characterize the structure of the optimal procurement policy under the lost-sales assumption. Several recent papers take into consideration financial hedging which complements a firm's operational decisions (see, e.g., Dong and Liu 2007, Kaminsky et al. 2008, Kouvelis et al. 2013). In the literature on inventory management with spot market price fluctuation and multisourcing, the sales price is treated as an exogenous constant or random variable. Our paper, however, endogenizes the pricing decision of the firm. This new treatment enables us to deliver new insights on the pricing behavior of the firm under

procurement cost fluctuation: (a) the firm should pass the cost fluctuation to its down stream customers through dynamic pricing, and (b) dynamic pricing and dual sourcing may be either strategic complements or substitutes under demand uncertainty and procurement cost fluctuation.

This work is also related to the extensive literature on joint pricing and inventory management under stochastic demand. Petruzzi and Dada (1999) give a comprehensive review on the single period joint pricing and inventory control problem and extend the results in the newsvendor problem with pricing. Federgruen and Heching (1999) provide a general treatment of the dynamic pricing and inventory management problem. They show that a list-price/order-up-to policy is optimal. This line of literature has grown rapidly since Federgruen and Heching (1999). For example, Chen and Simchi-Levi (2004a, b, 2006) analyze the joint pricing and inventory control problem with fixed setup cost and show the optimality of (s, S, p) policy for finite horizon, infinite horizon, and continuous review models. Chen et al. (2006), Huh and Janakiraman (2008), and Song et al. (2009) study the joint pricing and inventory control problem under lost sales. In the case of a single unreliable supplier, Li and Zheng (2006) and Feng (2010) show that supply uncertainty drives the firm to charge higher prices under random yield and random capacity, respectively. When the replenishment lead time is positive, the joint pricing and inventory control problem under periodic review is extremely difficult, and Pang et al. (2012) partially characterize the structure of the optimal policy. We refer interested readers to Chen and Simchi-Levi (2012) for a comprehensive survey on joint pricing and inventory control models. This line of research assumes that the procurement costs are deterministic. Our paper incorporates procurement cost fluctuation into the standard joint pricing and inventory management model. As a result, we fully characterize the impact of procurement cost fluctuation upon the optimal pricing and inventory decisions, and we identify that the firm may earn higher expected profit under a more volatile procurement cost process.

From the modeling perspective, Zhou and Chao (2014) is the closest to this work. They study the value of dynamic pricing and dual sourcing with regular and expedited supplies to mitigate the demand uncertainty risk under *deterministic* procurement costs. They characterize the optimal policy as a state-dependent two-threshold list-price policy. Our focus is on how procurement cost fluctuation influences the optimal pricing and sourcing policy. More specifically, the driving force for dual sourcing in Zhou and Chao (2014) is the cost-and-lead-time trade-off between the regular and expedited supplies, whereas

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in our model, dual sourcing is driven by procurement cost fluctuation. Our model offers new insights on the impact of procurement cost fluctuation: (a) we analytically show that a risk-neutral firm may earn higher expected profit under a more volatile spot market cost process, and (b) we numerically show that procurement cost fluctuation may alter the strategic relationship between dynamic pricing and dual sourcing.

3. Model

We consider a T-period inventory system of a firm facing price-sensitive random demand. The periods are indexed backwards as $\{T, T-1, \ldots, 1\}$. The firm is a price setter and procures inventory via two channels: (a) directly from the spot market with fluctuating unit procurement cost, c_t; and (b) through the forward-buying contract written on the spot price, with unit cost $f_t = F_t(c_t)$, where $F_t(\cdot)$ is a positive increasing function of c_t. We assume that the lead time for the forward-buying contract is one period for tractability (see, e.g., Fudaka 1964, Whittmore and Saunders 1977, Feng et al. 2006). The objective of the firm is to determine the joint pricing and ordering policy that maximizes the total expected discounted profit over the planning horizon. The discount factor is denoted as $\alpha \in (0, 1)$.

In each period, the sequence of events unfolds as follows. At the beginning of period t, the inventory preordered in the last period through the forward-buying contract arrives, and the firm reviews its inventory level I_t and the realized spot market procurement cost c_t. The firm then makes a simultaneous decision of spot-purchasing quantity, $x_t - I_t$ (where x_t is the order-up-to level from the spot market), forward-buying quantity q_t, and sales price p_t. The order from the spot market is received immediately but through the forward-buying contract will be delivered at the beginning of the next period. The price-dependent stochastic demand $D_t(p_t)$ then realizes. The firm collects revenue from the realized demand. Unsatisfied demand is fully backlogged with unit backlogging cost b. Excess inventory is carried over to the next period with unit holding cost h. Note that although the parameters and demands are assumed to be stationary for expositional convenience, the structural results in this paper remain valid when they are time dependent.

Without loss of generality, we assume that the sales price p_t is bounded from above by the maximum allowable price $\bar{p}$ and bounded from below by the minimum allowable price $\underline{p}$. The demand in period t, $D_t(p_t)$, is stochastic and depends on the sales price p_t in the following functional form: $D_t(p_t) := d(p_t) + \epsilon_t$, where $d(\cdot)$ is a strictly decreasing function on $[\underline{p}, \bar{p}]$, and ϵ_t's are independent continuous random variables with support $[\underline{\epsilon}, \bar{\epsilon}]$ and $\mathbb{E}\{\epsilon_t\} = 0$. $d(p_t)$

refers to the expected demand in period t when the firm charges the sales price p_t, and summarizes the impact of sales price on demand. Since $d(\cdot)$ is strictly decreasing, it has a strictly decreasing inverse $p(\cdot)$ that maps from $[\underline{d}, \bar{d}]$ to $[\underline{p}, \bar{p}]$, where $\underline{d} = d(\bar{p})$ and $\bar{d} = d(\underline{p})$. To ensure that the demand in each period is nonnegative, we assume that $\underline{d} + \underline{\epsilon} \geq 0$. For the convenience of our analysis, we change the decision variable from the sales price p_t to the expected demand d_t. For technical tractability, we impose the following standard assumption throughout our analysis:

ASSUMPTION 1. $R(d_t) := p(d_t)d_t$ *is continuously differentiable and concave on the region* $[\underline{d}, \bar{d}]$.

Note that the strict monotonicity of $p(\cdot)$, together with the concavity of $R(\cdot)$, implies that $R(\cdot)$ is strictly concave. This demand model has been extensively employed in the joint pricing and inventory management literature with deterministic procurement costs (ee, e.g., Chen and Simchi-Levi 2004a, Pang et al. 2012, Zhou and Chao 2014).

The procurement cost process in the spot market $\{c_t\}_{t=T}^1$ evolves based on the following Markovian scheme: $c_{t-1} = s_t(c_t, \xi_t)$, where $s_t(\cdot, \cdot)$ is a positive valued function and ξ_t is a random perturbation term in the cost dynamics, for $t = T, T-1, \ldots, 1$. Assume that $\mathbb{E}\{s_t(c_t, \xi_t)\} < +\infty$. Assume for any $\hat{c}_t > c_t$, $s_t(\hat{c}_t, \xi_t) \geq_{s.d.} s_t(c_t, \xi_t)$, where $\geq_{s.d.}$ refers to first-order stochastic dominance; i.e., a higher current cost is more likely to give rise to higher future costs. We remark that the above cost model is quite general and consistent with the well-established and empirically justified commodity price models in the finance literature, such as geometric Brownian motions, where $s_t(c_t, \xi_t) = (c_t/\alpha)e^{\sigma B_t - \sigma^2/2}$, and mean-reverting processes, where $s_t(c_t, \xi_t) = (c_t/\alpha) \exp[\kappa(\mu - \log(c_t)) + \sigma B_t - \sigma^2/2]$, with $\sigma > 0$, $0 < \kappa < 1$ and $\{B_t\}$ following independent and identically distributed (i.i.d.) standard normal distributions (see, e.g., Fama and French 1988, Schwartz 1997, Geman 2005). Let $\mu_t(c_t) := \mathbb{E}\{s_t(c_t, \xi_t) \mid c_t\}$. It is clear that $\mu_t(\cdot)$ is increasing in the current spot-market purchasing cost c_t. We remark that in the perfect market under the risk-neutral probability measure, $\mu_t(c_t) = c_t/\alpha$, but this identity may not hold under a more general spot market price process. In consistency with the "partially complete" market assumption (see, e.g., Smith and Nau 1995, Kouvelis et al. 2013), we assume that, for any given c_t, ϵ_t is independent from $c_{t-1}, c_{t-2}, \ldots, c_1$. Moreover, we assume in our model that the firm cannot resell its excess inventory in the spot market, because of, e.g., (a) high operational and labor costs associated with

inventory reselling, (b) contractual restrictions, and (c) operational limitations (see, e.g., Chen et al. 2013). Hence, there is no room for arbitrage in our model.

In addition to the spot market, the firm also resorts to an alternative procurement channel: forward-buying contract. The forward-buying contract mitigates procurement cost volatility at the cost of reduced procurement responsiveness. In period t, the firm signs a forward-buying contract (f_t, q_t) with a supplier which prescribes that the firm pays the supplier $f_t q_t$ and the supplier delivers q_t units of the item at the beginning of period t^e. As discussed above, we assume that $t^e = t - 1$ for tractability; i.e., the lead time for the forward-buying contract is one period. In the perfect market under the risk-neutral probability measure, $f_t = \mu_t(c_t) = c_t/\alpha$ by the standard nonarbitrage argument. Equivalently, the cash flow is $-c_t q_t$ if the firm preorders q_t units in period t. In reality, the forward price f_t is determined through bilateral negotiations between the firm and the suppliers in each period based on the realized spot price c_t (see, e.g., Dong and Liu 2007, Nagali et al. 2008, Nystedt 2007). In our model, for simplicity and clarity, we do not explicitly model the negotiation process, but we assume that the resulting contract satisfies $f_t = \gamma c_t/\alpha$; i.e., the unit effective cost is γc_t, where $\gamma > 0$. If $\gamma < 1$, the firm receives a preorder discount, whereas if $\gamma > 1$, it is more expensive to order through the forward-buying contract. We remark that most of our results hold under the more general forward-buying contract with $f_t = F_t(c_t)$, where $F_t(\cdot)$ is a positive increasing function of the spot price c_t. We do not allow the forward-buying contract to be traded in the derivatives market, because our focus is on the operational implication of the forward-buying contract instead of its financial hedging effect.

We now formulate the planning problem as a dynamic program. The inventory dynamics of our model satisfy the following equality: $I_{t-1} = x_t + q_t - d_t - \epsilon_t$, where $x_t - I_t$ is the order quantity from the spot market, q_t is the order quantity through the forward-buying contract, and $p(d_t)$ is the sales price in period t. Let $V_t(I_t \mid c_t)$ denote the maximum expected discounted total profit in periods $t, t-1, \ldots, 0$, when the starting inventory level in period t is I_t and the realized procurement cost is c_t. In the last period, no forward-buying contract is signed, i.e., $q_1 = 0$. Without loss of generality, we assume that excess inventory in the last period is discarded without salvage value, i.e., $V_0(I_0 \mid c_0) = 0$. The optimal value functions satisfy the following recursive scheme:

$$V_t(I_t \mid c_t) = c_t I_t + \max_{x_t \geq I_t,\, q_t \in \mathbb{Q}_t,\, d_t \in [\underline{d}, \bar{d}]} J_t(x_t, q_t, d_t \mid c_t),$$

$$\text{where } \mathbb{Q}_t := \begin{cases} [0, +\infty] & \text{if } t \geq 2, \\ \{0\} & \text{otherwise,} \end{cases} \quad \text{and}$$

$$\begin{aligned}
J_t(x_t, q_t, d_t \mid c_t) &= -c_t I_t + \mathbb{E}\{p(d_t)D_t - c_t(x_t - I_t) - \gamma c_t q_t \\
&\quad - h(x_t - D_t)^+ - b(x_t - D_t)^- \\
&\quad + \alpha V_{t-1}(x_t + q_t - D_t \mid s_t(c_t, \xi_t)) \mid c_t\} \\
&= (p(d_t) - b - \alpha \mu_t(c_t))d_t + (b - c_t \\
&\quad + \alpha \mu_t(c_t))x_t + (\alpha \mu_t(c_t) - \gamma c_t)q_t \\
&\quad + \mathbb{E}\{-(h + b)(x_t - d_t - \epsilon_t)^+] \\
&\quad + \alpha[V_{t-1}(x_t + q_t - d_t - \epsilon_t \mid s_t(c_t, \xi_t)) \\
&\quad - s_t(c_t, \xi_t)(x_t + q_t - d_t - \epsilon_t)] \mid c_t\} \\
&= R(d_t \mid c_t) + (b - c_t + \alpha \mu_t(c_t))x_t \\
&\quad + (\alpha \mu_t(c_t) - \gamma c_t)q_t + L(x_t - d_t) \\
&\quad + H_t(x_t + q_t - d_t \mid c_t),
\end{aligned}$$

with $R(d_t \mid c_t) := (p(d_t) - b - \alpha \mu_t(c_t))d_t$, $L(y) := \mathbb{E}[-(b + h)(y - \epsilon_t)^+]$, and $H_t(y \mid c_t) := \alpha \mathbb{E}[V_{t-1}(y - \epsilon_t \mid s_t(c_t, \xi_t)) - s_t(c_t, \xi_t)(y - \epsilon_t) \mid c_t]$.

Therefore, for each period t, the firm reviews its starting inventory level I_t and spot-market procurement cost c_t, and chooses a joint pricing and replenishment policy $(x_t^*(I_t, c_t), q_t^*(I_t, c_t), d_t^*(I_t, c_t)) \in F(I_t)$ to maximize $J_t(x_t, q_t, d_t \mid c_t)$, where $F(I_t) := [I_t, +\infty] \times \mathbb{Q}_t \times [\underline{d}, \bar{d}]$. In the case of multiple maximizers, $(x_t^*(I_t, c_t), q_t^*(I_t, c_t), d_t^*(I_t, c_t))$ is defined to be the lexicographically smallest one. For any (I_t, c_t), we use $\Delta_t^*(I_t, c_t) := x_t^*(I_t, c_t) - d_t^*(I_t, c_t)$ to denote the optimal safety stock in period t.

To characterize the optimal joint pricing and inventory control policy under fluctuating procurement costs, we first establish the concavity and differentiability of the objective functions under Assumption 1 in the following lemma:

LEMMA 1. *For each* $t = T, T-1, \ldots, 1$ *and any given* c_t, *the following statements hold*:

(a) $H_t(y \mid c_t)$ *is concave, decreasing, and continuously differentiable in* y.

(b) $J_t(x_t, q_t, d_t \mid c_t)$ *is jointly concave and continuously differentiable in* (x_t, q_t, d_t).

(c) $V_t(I_t \mid c_t)$ *is finite, concave, and continuously differentiable.* $V_t(I_t \mid c_t) - c_t I_t$ *is decreasing in* I_t.

The proofs of Lemma 1 and all other technical results are relegated to the online supplement (available as supplemental material at http://dx.doi.org/10.1287/msom.2015.0519). Since $J_t(x_t, q_t, d_t \mid c_t)$ is jointly concave, we define the optimal unconstrained maximizer as

$$(x_t(c_t), q_t(c_t), d_t(c_t)) := \operatorname*{arg\,max}_{x_t \in \mathbb{R},\, q_t \in \mathbb{Q}_t,\, d_t \in [\underline{d}, \bar{d}]} J_t(x_t, q_t, d_t \mid c_t),$$

where, in the case of multiple maximizers, the lexicographically smallest one is selected. We use

$\Delta_t(c_t) := x_t(c_t) - d_t(c_t)$ to denote the optimal inventory-independent safety stock in period t with spot-market purchasing cost c_t. With the help of the above lemma, we now characterize the structure of the optimal dynamic-pricing/dual-sourcing policy in the following theorem:

THEOREM 1. *For each $t = T, T-1, \ldots, 1$, the following statements hold:*

(a) *If the starting inventory level $I_t < x_t(c_t)$, it is optimal to procure $x_t(c_t) - I_t$ in the spot market, sign a forward-buying contract $(\gamma c_t/\alpha, q_t(c_t))$, and charge a list-price $p(d_t(c_t))$; i.e., $(x_t^*(I_t, c_t), q_t^*(I_t, c_t), d_t^*(I_t, c_t)) = (x_t(c_t), q_t(c_t), d_t(c_t))$.*

(b) *In the remaining case, i.e., $I_t \geq x_t(c_t)$, it is optimal to order nothing from the spot market, i.e., $x_t^*(I_t, c_t) = I_t$, and $(q_t^*(I_t, c_t), d_t^*(I_t, c_t)) = \arg\max_{q_t \in \mathcal{Q}_t, d_t \in [\underline{d}, \bar{d}]} J_t(I_t, q_t, d_t \mid c_t)$.*

(c) *If $q_t(c_t) = 0$, it is optimal not to place any order with the forward-buying contract, regardless of the starting inventory level I_t, i.e., $q_t^*(I_t, c_t) \equiv 0$, and we define $I_t^*(c_t) := -\infty$. Otherwise, $q_t(c_t) > 0$, there exists a threshold inventory level $I_t^*(c_t) \geq x_t(c_t) + q_t(c_t)$, such that it is optimal to place an order with the forward-buying contract if and only if the starting inventory level is below the threshold, i.e.,*

$$q_t^*(I_t, c_t) \begin{cases} > 0 & I_t < I_t^*(c_t), \\ = 0 & \text{otherwise}. \end{cases}$$

(d) *$q_t^*(I_t, c_t)$ is continuously decreasing in I_t, whereas $d_t^*(I_t, c_t)$, $\Delta_t^*(I_t, c_t)$, and $x_t^*(I_t, c_t) + q_t^*(I_t, c_t)$ are continuously increasing in I_t.*

Lemma 1 and Theorem 1 generalize Theorems 1 and 2 in Federgruen and Heching (1999) and Lemma 1 and Theorem 4 in Zhou and Chao (2014) to the model with fluctuating spot-purchasing costs. Specifically, we show that the optimal policy is a cost-dependent order-up-to/preorder-up-to/list-price policy with two ordering thresholds, $x_t(c_t)$ and $I_t^*(c_t)$. Consistent with Zhou and Chao (2014), we prove that the optimal forward-buying quantity, $q_t^*(I_t, c_t)$, and the optimal sales price, $p(d_t^*(I_t, c_t))$, are decreasing, whereas the optimal safety stock, $\Delta_t^*(I_t, c_t)$, and the optimal total order-up-to level, $x_t^*(I_t, c_t) + q_t^*(I_t, c_t)$, are increasing, in the starting inventory level I_t.

Next, we examine which sourcing channel the firm should adopt. The following theorem characterizes sufficient conditions under which it is optimal not to source both from the spot market and through the forward-buying contract.

THEOREM 2. (a) *For $t = T, T-1, \ldots, 2$, if $b \leq \max\{c_t - \gamma c_t, c_t - \alpha\mu_t(c_t)\}$, $x_t(c_t) = -\infty$.*

(b) *For $t = T, T-1, \ldots, 2$, if $\gamma c_t \geq \alpha\mu_t(c_t)$, $q_t^*(I_t, c_t) = 0$.*

(c) *For $t = 1$, $x_t(c_t) = -\infty$, if and only if $b - c_t \leq 0$.*

Part (a) of Theorem 2 establishes a sufficient condition under which it is optimal for the firm not to source from the spot market: the backlogging cost is dominated by the cost saving from the forward-buying contract or from postponing the procurement in the spot market to the next period. Moreover, part (b) gives a sufficient condition under which it is optimal not to procure through the forward-buying contract: the unit procurement cost with the forward-buying contract exceeds the unit expected procurement cost from the spot market in the next period. Theorem 2(b) implies that the sole-sourcing model with spot-market purchase can be viewed as a special case of our dual-sourcing model by setting $\gamma \geq \sup_{t, c_t}\{(\alpha\mu_t(c_t))/c_t\}$. Similarly, by setting $b \leq \inf_{t, c_t}\{\max\{c_t - \gamma c_t, c_t - \alpha\mu_t(c_t)\}\}$, we obtain the sole-sourcing model with forward-buying purchase as a special case.

4. Impact of Procurement Cost Fluctuation

Our model is driven by demand uncertainty and procurement cost fluctuation. Whereas the former driving force has been extensively studied in the joint pricing and inventory management literature, the latter has received little attention in this stream of research. One objective of this paper is to understand the impact of cost fluctuation upon the firm's profit, and its optimal pricing and sourcing decisions.

To begin with, we study how the procurement cost volatility influences the total expected profit of the firm. The following theorem shows that a risk-neutral profit-maximizing firm may earn higher expected profit in each period when the cost volatility is higher.

THEOREM 3. *For two procurement cost processes $\{c_t\}_{t=T}^1$ and $\{\hat{c}_t\}_{t=T}^1$, assume that for every $t = T, T-1, \ldots, 1$, $s_t(c_t, \xi_t)$ and $\hat{s}_t(c_t, \xi_t)$ are concavely increasing in c_t for any realization of ξ_t. The following statements hold:*

(a) *$V_t(I_t \mid c_t)$ is convexly decreasing in c_t, for any I_t.*

(b) *Assume that $\{c_t\}_{t=T}^1$ and $\{\hat{c}_t\}_{t=T}^1$ are identical except that $\hat{s}_\tau(c_\tau, \xi_\tau) \geq_{cx} s_\tau(c_\tau, \xi_\tau)$ for some c_τ and τ, where $\geq_{cx}$ refers to greater in convex order. $\hat{V}_t(I_t \mid c_t) \geq V_t(I_t \mid c_t)$ for each (I_t, c_t) and t, where $\{\hat{V}_t(I_t \mid c_t)\}_{t=T}^1$ are the value functions associated with $\{\hat{c}_t\}_{t=T}^1$.*

Theorem 3 is mainly driven by the timing of decision making with respect to uncertainty realization. More specifically, the firm makes its pricing and inventory decisions *posterior to* the cost realization in each period. In other words, the firm is blessed with the flexibility of *responding to* cost fluctuation with the information carried by the realized current period cost, which enables it to extract the high profit with low realized costs to compensate for the low profit

with high realized costs. The flexibility of responding to cost fluctuation renders that the value function $V_t(I_t \mid c_t)$ is convex in the procurement cost c_t and, hence, enables the firm to benefit from procurement cost volatility. Similar observation on the impact of decision timing with respect to uncertainty realization has been established in the capacity management and newsvendor network literature (see, e.g., Van Mieghem and Dada 1999, Chod and Rudi 2005, Bish et al. 2012), which shows that a firm with responsive pricing earns higher expected profit under more variable demands, because it can effectively respond to demand realization.

In the OM-finance interface literature, it is often argued that cost volatility deteriorates the utility of a firm so that hedging with financial instruments (e.g., options, futures, etc.) is often recommended (see, e.g., Kouvelis et al. 2013). Note that this stream of literature assumes that the firm is risk averse, and, as a result, the value and objective functions are concave in the procurement cost. Hence, the risk aversion of the firm may render Theorem 3 invalid. When the firm is risk seeking with convex intraperiod utility functions, Theorem 3 still holds with an analogous proof.

Theorem 3 needs the assumption that $s_t(c_t, \xi_t)$ is concavely increasing in c_t for any realization of ξ_t. This assumption is not restrictive and is satisfied by the commonly used commodity price models such as geometric Brownian motions and mean-reverting processes. Intuitively, this assumption means that the spot market price dynamics have the mean-reverting property, which is commonly observed in practice and empirically justified (see, e.g., Schwartz 1997). When this assumption does not hold, our numerical experiments in §6.3 demonstrate that the result in Theorem 3 is robust and remains valid for most of the initial procurement costs and, in particular, when the initial procurement cost follows the stationary distribution.

As discussed above, the firm generates profit from cost volatility by responding to the cost changes over the planning horizon. In the remainder of this section, we study *how* the firm dynamically adjusts its pricing and sourcing decisions in response to cost fluctuation. First, we analyze whether the firm should always pass the procurement cost fluctuation to its downstream customers.

THEOREM 4. *For* $t = T, T-1, T-2, \ldots, 1$, *assume that* $\hat{c}_t > c_t$. *The following statements hold:*

(a)
$$d_t(c_t) = \arg\max_{d_t \in [\underline{d}, \bar{d}]} \{R(d_t) - c_t d_t\}. \tag{1}$$

(b) $\partial_{I_t} V_t(I_t \mid \hat{c}_t) \geq \partial_{I_t} V_t(I_t \mid c_t)$ *for all* I_t.
(c) $d_t(\hat{c}_t) \leq d_t(c_t)$, $d_t^*(I_t, \hat{c}_t) \leq d_t^*(I_t, c_t)$, *and, thus,* $p(d_t^*(I_t, \hat{c}_t)) \geq p(d_t^*(I_t, c_t))$, *for all* I_t.

In Theorem 4(a), we show that the optimal list-price is determined myopically via optimizing the expected difference between revenue and procurement cost in the current period. Part (b) of Theorem 4 implies that the marginal value of inventory is increasing in the current procurement cost in each period. Part (c) indicates that in order to effectively address the procurement cost fluctuation, the firm should pass the cost fluctuation to its customers by adjusting the sales price accordingly. Moreover, the optimal sales price is increasing in the current procurement cost.

By increasing the sales price in response to high procurement cost, the firm may suffer from potential demand loss. Intuitively, the firm should decrease the spot-purchasing and forward-buying quantities to better match supply with demand. However, this intuition may not be true since current procurement cost also summarizes the information regarding the costs in future periods. Therefore, the firm should deal with both the trade-off between understock and overstock, and the risk and opportunity of fluctuating costs. With a higher current procurement cost, the firm may order less to save the current cost or order more to speculate in anticipation of higher future costs. Hence, the impact of cost fluctuation on the firm's optimal inventory policy is quite involved. To resolve this conundrum, we first rewrite the objective function in each decision period $J_t(\cdot, \cdot, \cdot \mid c_t)$ in terms of safety stock $\Delta_t := x_t - d_t$, forward-buying quantity q_t, and expected demand d_t as follows:

$$J_t(x_t, q_t, d_t \mid c_t) = R(d_t) - c_t x_t - \gamma c_t q_t + \Lambda(x_t - d_t)$$
$$+ \Psi_t(x_t + q_t - d_t \mid c_t) \tag{2}$$
$$= [R(d_t) - c_t d_t] + [\Lambda(\Delta_t) - (1-\gamma)c_t \Delta_t]$$
$$+ [\Psi_t(\Delta_t + q_t \mid c_t) - \gamma c_t(\Delta_t + q_t)] \tag{3}$$
$$=: \tilde{J}_t(\Delta_t, q_t, d_t \mid c_t),$$

where $\Lambda(y) := \mathbb{E}\{-h(y - \epsilon_t)^+ - b(y - \epsilon_t)^-\}$ and $\Psi_t(y \mid c_t) := \alpha \mathbb{E}_{\epsilon_t, \xi_t}\{V_{t-1}(y - \epsilon_t \mid s_t(c_t, \xi_t)) \mid c_t\}$. It is clear by the proof of Lemma 1 that $\Lambda(\cdot)$ and $\Psi_t(\cdot \mid c_t)$ are concave and continuously differentiable in y for any c_t. By Equation (3), the firm employs the dynamic pricing strategy to extract the current period profit (the first term), sets the safety stock to hedge against the current period demand uncertainty (the second term), and coordinates the ordering quantities from both channels (spot purchasing and forward buying) to speculate on the future procurement cost fluctuation (the last term). In particular, it can be viewed that the firm splits the current period spot market purchasing cost c_t into the sum of $(1-\gamma)c_t$ and γc_t, the former as the (hypothetical) ordering cost to hedge against the current period demand uncertainty, and the latter as the (hypothetical) ordering cost to speculate on the future cost fluctuation. The second term

in Equation (3) implies that an increase in the current period spot market purchasing cost c_t, may increase or decrease the (hypothetical) ordering cost, $(1 - \gamma)c_t$, depending on the magnitude of γ relative to 1. Thus, the dependence of the optimal safety stock upon the current spot-purchasing cost also relies on the value of γ, as shown in the following theorem.

THEOREM 5. *For* $t = T, T - 1, T - 2, \ldots, 1$, *assume that* $\hat{c}_t > c_t$. *The following statements hold:*

(a) *If* $q_t(c_t) > 0$ *and* $x_t(c_t) > -\infty$, $\Delta_t(c_t) = \arg\max_{\Delta_t}\{\Lambda(\Delta_t) - (1 - \gamma)c_t\Delta_t\}$.

(b) *If* $\gamma \le 1$ *and* $q_t(c_t) > 0$, $\Delta_t(c_t) \ge \Delta_t(\hat{c}_t)$, $x_t(c_t) \ge x_t(\hat{c}_t)$, *and* $x_t^*(I_t, c_t) \ge x_t^*(I_t, \hat{c}_t)$, *for all* I_t.

(c) *If* $\gamma > 1$ *and* $q_t(\hat{c}_t) > 0$, $\Delta_t(\hat{c}_t) \ge \Delta_t(c_t)$.

Theorem 5(a) shows that when it is optimal to order through both sourcing channels, the optimal safety stock is obtained by solving a standard single-period newsvendor model. Theorem 5(b) proves that whenever it is optimal to order through the forward-buying contract, if forward-buying is less expensive than spotpurchasing (i.e., $\gamma \le 1$), the optimal safety stock is decreasing in the current period spot market purchasing cost. Conversely, in Theorem 5(c), if spot purchasing is more expensive ($\gamma > 1$), the optimal safety stock is increasing in the current period spot market purchasing cost. This result contrasts with the finding in the current literature, which shows that, without the forward-buying procurement channel, the optimal order quantity and safety stock from the spot market are generally not monotone in the current spot market price (see, e.g., Li et al. 2001). Hence, our model delivers the new insight that the introduction of the forward-buying contract changes the sourcing behavior of the firm in the spot market.

As demonstrated by Theorem 5, the relative cost between spot purchasing and forward buying may significantly alter the firm's response to spot-purchasing cost changes. If spot purchasing is more expensive ($\gamma \le 1$), it is cost effective to split the goal of hedging against current period demand uncertainty risk and speculating on future cost fluctuation, the former with spot purchasing and the latter with forward buying. In this case, an increase in the spot-purchasing cost c_t increases the overage cost to hedge against current period demand uncertainty, and hence decreases the optimal safety stock and optimal order-up-to level from the spot market. On the other hand, however, if forward buying is more expensive ($\gamma > 1$), safety stock also plays an important role in speculating on future cost fluctuation. In this case, an increase in c_t leads to more speculation on higher future procurement costs, thus prompting the firm to set a higher safety stock.

To understand the impact of current cost on the optimal forward-buying quantity, we define the

expected discounted cost increment between period t and period $t - 1$ given the current cost c_t, $\kappa_t(c_t) := \alpha\mu_t(c_t) - c_t$. If $\kappa_t(c_t)$ is decreasing in c_t, the spot price grows more rapidly at a lower current cost. We have the following theorem on the impact of current procurement cost upon the optimal forward-buying quantity.

THEOREM 6. *For* $t = T, T - 1, T - 2, \ldots, 1$, *assume that* $\hat{c}_t > c_t$, $\gamma = 1$, *and* $\kappa_s(\cdot)$ *is a decreasing function of* c_s, *for any* $s = T, T - 1, \ldots, 1$. *The following statements hold:*

(a) $\partial_{I_t} V_t(I_t \mid \hat{c}_t) - \hat{c}_t \le \partial_{I_t} V_t(I_t \mid c_t) - c_t$.

(b) $\Delta_t(\hat{c}_t) \le \Delta_t(c_t)$, $x_t(\hat{c}_t) \le x_t(c_t)$, *and* $I_t^*(\hat{c}_t) \le I_t^*(c_t)$.

(c) $q_t^*(I_t, \hat{c}_t) \le q_t^*(I_t, c_t)$ *for all* I_t, *and, in particular,* $q_t(\hat{c}_t) \le q_t(c_t)$.

Theorem 6 shows that when the procurement cost grows more rapidly at a lower cost level (i.e., $\kappa_t(c_t)$ is decreasing in c_t) and the two sourcing channels have the same cost ($\gamma = 1$), the normalized marginal value of inventory is decreasing in the procurement cost, and the speculation opportunity does not justify the procurement cost increase. Therefore, with a higher current procurement cost, the firm should order less both from the spot market and through the forward-buying contract, and maintain a lower safety stock. Theorem 6 is consistent with some other results in the literature on commodity inventory management with sole-sourcing in the spot market, which characterize the monotonicity of order-up-to levels in the spot market price under the assumption that $\kappa_t(c_t)$ is decreasing in c_t (see, e.g., Theorem 5 of Kaminsky et al. 2008).

On the other hand, if $\kappa_t(c_t)$ is not decreasing in c_t, the expected cost increase may be higher at a higher cost level, and the firm may invest more on inventory through forward buying to speculate when the current cost is higher. In this case, the optimal forward-buying quantity may not be monotone in the current procurement cost, as shown by a numerical example in the online supplement (Figure 3).

In addition to the current procurement cost, the firm should also take into account the future cost trend to better exploit the cost fluctuation, as shown in the following theorem:

THEOREM 7. *Let two inventory systems be equivalent in everything except that there exists a* t_*, *such that* $\hat{s}_{t_*}(c_{t_*}, \xi_{t_*}) \ge_{s.d.} s_{t_*}(c_{t_*}, \xi_{t_*})$ *for every* c_{t_*}. *Let* $\{\hat{V}_t(I_t \mid c_t)\}_{t=T}^1$ *denote the value functions associated with* $\{\hat{c}_t\}_{t=T}^1$. *For* $t \ge t_*$ *and any* c_t, *the following statements hold:*

(a) $\partial_{I_t}\hat{V}_t(I_t \mid c_t) \ge \partial_{I_t}V_t(I_t \mid c_t)$, *for* $t \ge t_* + 1$ *and any* I_t.

(b) $\hat{q}_t(c_t) \ge q_t(c_t)$, *and* $\hat{q}_t^*(I_t, c_t) \ge q_t^*(I_t, c_t)$ *for all* I_t. $\hat{I}_t^*(c_t) \ge I_t^*(c_t)$.

(c) $\hat{x}_t(c_t) \ge x_t(c_t)$, *and* $\hat{x}_t^*(I_t, c_t) \ge x_t^*(I_t, c_t)$ *for all* I_t. *If* $q_t(c_t) > 0$, $\hat{x}_t(c_t) = x_t(c_t)$.

(d) $\hat{d}_t(c_t) = d_t(c_t)$, and $\hat{d}_t^*(I_t, c_t) \le d_t^*(I_t, c_t)$ for all I_t.

(e) $\hat{\Delta}_t(c_t) \ge \Delta_t(c_t)$, and $\hat{\Delta}_t^*(I_t, c_t) \ge \Delta_t^*(I_t, c_t)$ for all I_t.

Theorem 7 shows that under a higher procurement cost trend, the marginal value of inventory increases, so the firm should increase its spot-purchasing and forward-buying quantities, and set higher safety stocks and sales prices. In particular, the dual-sourcing strategy grants the firm the flexibility to separate the inventory for current period and that for future periods. Therefore, the firm responds to a higher procurement cost trend by increasing its order quantity through the forward-buying contract. The order quantity from the spot market, however, should stay the same, unless the firm sources from the spot market alone (i.e., $q_t(c_t) = 0$). We remark that the monotonicity results in Theorem 7 can be easily generalized to the case in which one system has higher cost trends in multiple periods.

5. Impact of Dual-Sourcing Flexibility

As discussed above, the dual-sourcing strategy enables the firm to benefit from the portfolio effect of the more responsive spot-purchasing channel with more volatile cost and the less responsive forward-buying channel with more predictable cost. In this section, we study how dual-sourcing flexibility impacts the optimal policy of the firm.

Recall that γ measures the procurement cost through the forward-buying contract relative to the spot market price: the smaller the γ, the cheaper the forward-buying sourcing channel, and the higher the dual-sourcing flexibility. It is shown by Theorem 5 that the relative magnitude of γ to 1 determines the role of spot purchasing in speculating on future cost fluctuation and, thus, the firm's response to a higher procurement cost. Now we analyze how dual-sourcing flexibility (captured by γ) directly impacts the optimal pricing and sourcing decisions. To investigate such impact, we incorporate γ as a subscript of the value function $V_{\gamma,t}(\cdot \mid \cdot)$, the objective function $J_{\gamma,t}(\cdot, \cdot, \cdot \mid \cdot)$, and the optimal decision variables $(x_{\gamma,t}(\cdot), q_{\gamma,t}(\cdot), d_{\gamma,t}(\cdot))$, $(x_{\gamma,t}^*(\cdot, \cdot), q_{\gamma,t}^*(\cdot, \cdot), d_{\gamma,t}^*(\cdot, \cdot))$, $\Delta_{\gamma,t}(\cdot)$, and $\Delta_{\gamma,t}^*(\cdot, \cdot)$. The following theorem characterizes the impact of dual-sourcing flexibility upon the optimal pricing and inventory decisions.

THEOREM 8. *For $t = T, T-1, \dots, 2, 1$, $\hat{\gamma} > \gamma$, the following statements hold for any (I_t, c_t):*

(a) $\partial_{I_t} V_{\hat{\gamma},t}(I_t \mid c_t) \ge \partial_{I_t} V_{\gamma,t}(I_t \mid c_t)$.

(b) $\Delta_{\hat{\gamma},t}(c_t) \ge \Delta_{\gamma,t}(c_t)$, $\Delta_{\hat{\gamma},t}^*(I_t, c_t) \ge \Delta_{\gamma,t}^*(I_t, c_t)$, $x_{\hat{\gamma},t}(c_t) \ge x_{\gamma,t}(c_t)$ and $x_{\hat{\gamma},t}^*(I_t, c_t) \ge x_{\gamma,t}^*(I_t, c_t)$.

(c) $d_{\hat{\gamma},t}(c_t) = d_{\gamma,t}(c_t)$ and $d_{\hat{\gamma},t}^*(I_t, c_t) \le d_{\gamma,t}^*(I_t, c_t)$.

(d) *If $\kappa_s(c_s) \le 0$ for all $1 \le s \le T$ and c_s, $\partial_{I_t} V_{\hat{\gamma},t}(I_t \mid c_t) - \hat{\gamma} c_t \le \partial_{I_t} V_{\gamma,t}(I_t \mid c_t) - \gamma c_t$, $q_{\hat{\gamma},t}(c_t) \le q_{\gamma,t}(c_t)$ and $q_{\hat{\gamma},t}^*(I_t, c_t) \le q_{\gamma,t}^*(I_t, c_t)$.*

Theorem 8 demonstrates that, when the dual-sourcing flexibility is higher (lower γ), the firm sources less inventory from the more risky spot market and less intensively passes the procurement cost risk to its customers. Theorem 8(d) proves that when the effective expected procurement cost is decreasing over the planning horizon (i.e., $\kappa_t(c_t) \le 0$ for all t and c_t), the firm should order more through the forward-buying contract when the dual-sourcing flexibility is higher. We remark that the optimal forward-buying quantity $q_{\gamma,t}^*(\cdot, \cdot)$, contrary to our intuition, may not be monotone in γ. A lower dual-sourcing flexibility (i.e., bigger γ) increases not only the current procurement cost through the forward-buying contract, but also the marginal value of future inventory. When $\kappa_t(c_t)$ is positive, the latter effect may dominate the former effect, and it may be optimal for the firm to order more through the forward-buying contract when γ is bigger.

As shown by Theorem 2(b), if $\gamma \ge \sup_{t, c_t}\{(\alpha\mu_t(c_t))/c_t\}$, the firm will not place any order through the forward-buying contract, and the model is reduced to one with a sole-sourcing firm, which replenishes inventory from the spot market only. By Theorem 8, compared with sourcing from both channels, the firm orders more from the spot market, maintains a higher safety stock, and charges a higher sales price when only spot purchasing is allowed. This is consistent with Theorem 5 in Zhou and Chao (2014), which proves the same result under deterministic procurement costs.

6. Numerical Studies

This section reports a set of numerical studies on (a) the strategic relationship between dynamic pricing and dual sourcing and (b) the robustness of Theorem 3.

6.1. Numerical Settings

Throughout our numerical studies, we assume that the expected demand is linear in price: $d(p_t) = a - kp_t$, where we normalize the market size $a = 1$ and the price sensitivity of demand $k = 1$. The random component of D_t, ϵ_t, follows i.i.d. normal distribution with mean 0 and variance 0.04. We restrict the feasible price range to $[0.2, 0.8]$, such that the maximum expected demand is $\bar{d} = 0.8$ and the minimum expected demand is $\underline{d} = 0.2$. We set $\alpha = 0.99$ and $\gamma = 0.95$. The holding cost is $h = 0.04$, and the backlogging cost is $b = 0.6$.

We assume that the procurement cost is driven by a stationary Markov chain with state space $C = \{c_i: c_i = 0.05 + 0.045i, 0 \le i \le 20, i \in \mathbb{Z}\}$. We use P to denote the transition probability matrix of the cost process, where P_{ij} is the probability that the cost in the current period is c_j given that the cost in the previous

period is c_i. The entry values of P_{ij} will be given in the specific experiments in §§6.2 and 6.3.

We evaluate the optimal profit under six different pricing and sourcing schemes: (a) V^{ss} (static pricing and sole sourcing from the spot market alone), (b) V^{ds} (dynamic pricing and sole sourcing from the spot market alone), (c) V^{sf} (static pricing and sole sourcing through the forward-buying contract alone), (d) V^{df} (dynamic pricing and sole sourcing through the forward-buying contract alone), (e) V^{sd} (static pricing and dual sourcing) and (f) V^{dd} (dynamic pricing and dual sourcing, $V_t(\cdot \mid \cdot)$ in our model). V^{ds}, V^{df}, and V^{dd} are obtained by solving the corresponding dynamic programs in our model with sole sourcing from the spot market, sole sourcing through the forward-buying contract, and dual sourcing, respectively. To compute V^{ss}, V^{sf}, and V^{sd}, we first evaluate the total profit for any fixed p_t in each t in the dynamic programs with sole sourcing from the spot market, sole sourcing through the forward-buying contract, and dual sourcing, and then maximize over p_t to obtain the optimal static pricing policies p^{ss}, p^{sf}, and p^{sd}, respectively. We take $I_T = 0$ as the reference initial inventory level in the evaluation of V^{ss}, V^{sf}, V^{ds}, V^{df}, V^{sd}, and V^{dd}.

6.2. Strategic Relationship Between Dynamic Pricing and Dual Sourcing

In this subsection, we numerically characterize the strategic relationship between dynamic pricing and dual sourcing under demand uncertainty and procurement cost fluctuation. Dynamic pricing mitigates demand uncertainty risk and exploits procurement cost fluctuation, because it not only passes the cost uncertainty to downstream customers but also controls demand to consume inventory in a more profitable way. On the other hand, dual sourcing improves the operational flexibility and addresses the procurement cost fluctuation. It is of both theoretical and practical interests to analyze the strategic relationship between these two methods under both demand uncertainty and procurement cost fluctuation.

Specifically, we examine the strategic relationship between dynamic pricing and dual sourcing under two scenarios: (a) the sole-sourcing firm procures inventory from the *spot market* only, and (b) the

sole-sourcing firm procures inventory through the *forward-buying contract* only. In scenario (a), the strategic relationship between pricing and sourcing flexibilities is characterized by the sign of $\lambda_1 := V^{dd} + V^{ss} - V^{sd} - V^{ds}$. Analogously, in scenario (b), define $\lambda_2 := V^{dd} + V^{sf} - V^{sd} - V^{df}$. If $\lambda_i \geq 0$ ($i = 1, 2$), dynamic pricing and dual sourcing are strategic complements. Otherwise, they are strategic substitutes. The transition probability matrix that defines the procurement cost process in this experiment is given in the online supplement.

Our extensive numerical experiments show that when the sole-sourcing firm procures inventory only from the spot market, dynamic pricing and dual sourcing are strategic complements, i.e., $\lambda_1 \geq 0$ in all of the experiments we have examined. Table 1 presents the results of a subset of the numerical experiments we have conducted. Compared with sourcing from the spot market alone, the firm under the dual-sourcing strategy sources part of its inventory through the less responsive forward-buying contract, so the flexibility to control demand via pricing becomes more valuable. In this scenario, demand uncertainty decreases the value of dual sourcing, which can be compensated by the improved supply-demand match generated by the dynamic pricing strategy. Therefore, dynamic pricing and dual sourcing are strategic complements under both demand uncertainty and procurement cost fluctuation.

In scenario (b) (i.e., the sole-sourcing firm procures through the forward-buying contract only), our numerical results (see Table 2) show that dynamic pricing and dual sourcing are strategic substitutes for most cases. In this scenario, the dual-sourcing strategy improves the supply-demand match by adding the more responsive spot-purchasing channel to the sourcing portfolio. Such responsiveness is less valuable in the presence of dynamic pricing, which controls demand to reduce the supply-demand mismatch. Hence, the pricing and sourcing flexibilities are strategic substitutes. However, exceptions may occur (i.e., $\lambda_2 > 0$) for some cases with initial costs under which the expected cost increase is high (i.e., large $\kappa_t(c_t)$). With such initial costs, the firm over orders to speculate on high future costs, and the dual-sourcing strategy amplifies the overage risk. Therefore, under the dual-sourcing strategy, the flexibility of controlling

Table 1   Strategic Relationship Between Dynamic Pricing and Dual Sourcing: Scenario (a)

T	$c_T = 0.05$	$c_T = 0.185$	$c_T = 0.32$	$c_T = 0.455$	$c_T = 0.59$	$c_T = 0.725$	$c_T = 0.86$	$c_T = 0.95$
10	0.063	0.040	0.025	0.014	0.006	0.001	0.001	0.001
20	0.082	0.062	0.046	0.032	0.021	0.014	0.012	0.011
30	0.098	0.078	0.061	0.047	0.036	0.029	0.027	0.025
40	0.111	0.091	0.074	0.060	0.049	0.042	0.040	0.039
50	0.123	0.103	0.086	0.072	0.061	0.054	0.052	0.051

Table 2   Strategic Relationship Between Dynamic Pricing and Dual Sourcing: Scenario (b)

T	$c_T = 0.05$	$c_T = 0.185$	$c_T = 0.32$	$c_T = 0.455$	$c_T = 0.59$	$c_T = 0.725$	$c_T = 0.86$	$c_T = 0.95$
10	0.021	0.017	−0.003	−0.017	−0.022	−0.023	−0.026	−0.028
20	0.006	0.004	−0.014	−0.026	−0.029	−0.029	−0.031	−0.034
30	−0.004	−0.005	−0.022	−0.034	−0.037	−0.037	−0.039	−0.041
40	−0.012	−0.013	−0.030	−0.041	−0.045	−0.044	−0.046	−0.048
50	−0.019	−0.020	−0.037	−0.048	−0.051	−0.051	−0.053	−0.055

the demand process to turn over the excess inventory via dynamic pricing becomes more valuable. Thus, dynamic pricing and dual sourcing can be strategic complements in scenario (b). This result contrasts with the numerical study in Zhou and Chao (2014), which concludes that these two strategies are substitutes if the sole-sourcing firm procures from the less responsive regular supply. Therefore, our model delivers the new insight that procurement cost fluctuation may alter the strategic relationship between dynamic pricing and dual sourcing.

To sum up, under demand uncertainty and cost fluctuation, dynamic pricing and dual sourcing may be either strategic complements or substitutes. Dynamic pricing mitigates demand risk and exploits cost fluctuation, whereas the additional sourcing channel in the dual-sourcing portfolio may intensify or dampen demand risk. In scenario (a), by adding the less responsive forward-buying sourcing channel, dual sourcing addresses cost fluctuation but intensifies demand risk, so dynamic pricing and dual-sourcing are strategic complements. In scenario (b), by adding the more responsive spot-market sourcing channel, dual sourcing dampens demand risk and exploits cost fluctuation, so dynamic pricing and dual sourcing are strategic substitutes, except for the cases where high expected cost increase induces significant overordering under the dual-sourcing strategy.

6.3. Impact of Procurement Cost Volatility

In this subsection, we numerically examine whether the expected profit of the firm is higher under a more volatile procurement cost process, when $s_t(c_t, \xi_t)$ is not necessarily concave in c_t. Our extensive numerical experiments confirm that Theorem 3 is robust and holds for most cases.

Now we give a numerical example to illustrate the robustness of Theorem 3. We consider three cost processes defined on C: $\{\{c_t\}, \{\hat{c}_t\}, \{\hat{\hat{c}}_t\}\}$, where $\hat{\hat{s}}_t(c_t, \xi_t) \geq_{cx} \hat{s}_t(c_t, \xi_t) \geq_{cx} s_t(c_t, \xi_t)$, for each c_t, and t. We give the transition probability matrix for each cost process in the online supplement. Figure 1 shows that $\mu_t(c_t)$ is not concave in c_t, and hence $\hat{\hat{s}}_t(\cdot, \xi_t)$, $\hat{s}_t(\cdot, \xi_t)$ and $s_t(\cdot, \xi_t)$ are not necessarily concave in c_t for some realization of ξ_t. Since the standard deviation of the future procurement cost is identical when conditioned on each current cost in each period, we use

the standard deviation to represent the corresponding cost process. Clearly, $\text{std}(\hat{\hat{s}}_t(c_t, \xi_t)) > \text{std}(\hat{s}_t(c_t, \xi_t)) > \text{std}(s_t(c_t, \xi_t))$ for each c_t. In this example, the firm earns higher expected total profit when the procurement cost process is more volatile for all initial spot-purchasing costs. Figure 2 plots the optimal profit V^{dd} for each procurement cost process with six initial spot-purchasing costs.

In our extensive numerical experiments, when the initial cost is drawn from the stationary distribution of the procurement cost process, the firm always earns higher expected profit under a more volatile procurement cost process; i.e., Theorem 3 holds in the expectation sense. With an arbitrarily given initial cost, Theorem 3 holds for most cases, but exceptions may occur if the initial procurement cost is low and $s_t(c_t, \xi_t)$ is not necessarily concave in c_t. This is because, when the initial procurement cost is low, a less volatile cost process stays at low costs for a long time with higher probability, so it may generate higher expected total profit. When $s_t(c_t, \xi_t)$ is concave in c_t, the trajectory of the cost process, with high probability, grows fast at a low initial cost even when the cost volatility is low. This drives the phenomenon that a more volatile procurement cost process generates higher expected total profit for *all* initial procurement costs, as shown by Theorem 3. In summary, our numerical experiments complement the insights

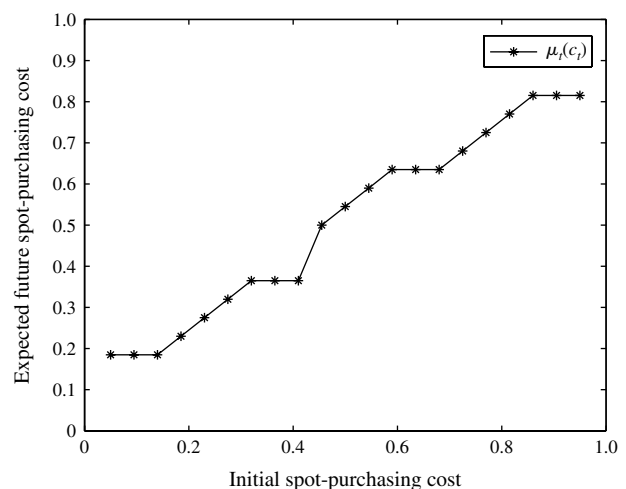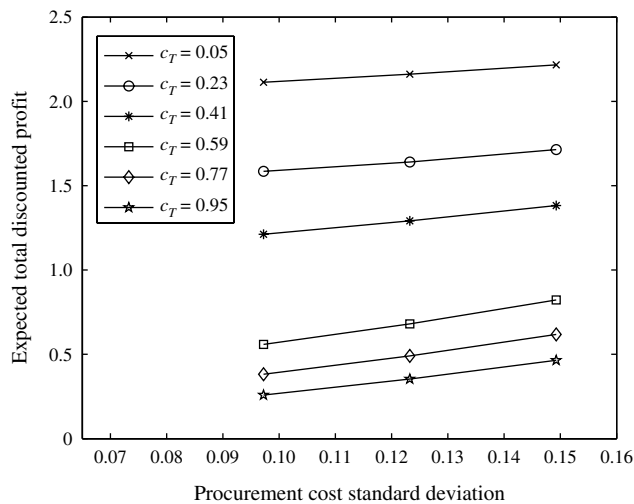**Figure 1   Expected Spot-Purchasing Cost in the Next Period**

Figure 2   Impact of Procurement Cost Volatility $(T = 20)$

of Theorem 3 and verify that the result of Theorem 3 is robust when $s_t(c_t, \xi_t)$ is not necessarily concave in c_t.

To conclude this section, we remark that we have also performed extensive numerical experiments to quantify the values of dynamic pricing and dual sourcing in the presence of demand uncertainty and cost fluctuation. Compared with the results in Zhou and Chao (2014), the values of these two strategies are, in general, significantly higher under procurement cost fluctuation than under deterministic costs. This difference demonstrates the effectiveness of dynamic pricing and dual sourcing in exploiting procurement cost fluctuation. Moreover, in Zhou and Chao (2014), the value of dual sourcing is generally higher than that of dynamic pricing. This observation is consistent with the numerical study in Federgruen and Heching (1999), which shows that the value of dynamic pricing is, in general, moderate, and it diminishes to zero as the planning horizon goes to infinity. In our numerical experiments, however, the value of dynamic pricing is generally much higher than that of dual sourcing, and it remains significant as the planning horizon extends, because the firm with static pricing cannot make price changes in response to cost fluctuation.

7.   Concluding Remarks

7.1.   Summary
This paper studies the joint pricing and inventory management model under fluctuating procurement costs and dual sourcing. To effectively hedge against demand uncertainty risk and exploit procurement cost fluctuation, the firm dynamically adjusts the sales price and replenishes its inventory either directly from the spot market or through the forward-buying contract. Our study investigates the crucial role of dynamic pricing and dual sourcing, and their strategic relationship, under demand uncertainty and cost fluctuation.

One focus of this paper is to study the impact of procurement cost fluctuation upon the firm's profit and optimal pricing and inventory decisions. Our analysis offers the unique insight that under a mild condition, a risk-neutral profit-maximizing firm earns higher expected profit under a more volatile procurement cost process. Thus, the cost volatility creates more opportunities than risks. This counterintuitive result is mainly driven by the timing of decision making with respect to uncertainty realization. The firm makes decisions in *response* to the cost realization in each period. Such responsiveness of decision making renders that the firm can effectively react to cost volatility by extracting the high profit with low realized costs to compensate for the low profit with high realized costs, thus profiting from the cost volatility.

We characterize how the firm should dynamically adjust its pricing and sourcing decisions in response to cost fluctuation. We show that the optimal sales prices increase in the current procurement cost. The impact of current procurement cost upon the optimal inventory decision, however, is more involved. The relative cost between forward buying and spot purchasing is crucial in understanding the optimal response to current cost changes. When spot purchasing is more expensive, the firm responds to a higher current cost by decreasing the safety stock. On the other hand, if forward buying is more expensive, the optimal safety stock is increasing in the current cost. The optimal forward-buying quantity is not necessarily monotone in the current procurement cost.

We also characterize the impact of dual-sourcing flexibility upon the optimal pricing and sourcing decisions of the firm. Higher dual-sourcing flexibility (i.e., smaller ratio between the effective cost of the forward-buying contract and the spot-market purchasing cost) prompts the firm to order less from the spot market and decrease the sales price. Higher dual-sourcing flexibility drives down both the forward-buying cost and the value of future inventory, so the firm may increase or decrease its forward-buying quantity, depending on which effect dominates.

We conduct extensive numerical experiments to study the strategic relationship between dynamic pricing and dual sourcing. Our numerical results demonstrate that dynamic pricing mitigates demand risk and exploits cost fluctuation, whereas dual sourcing may either intensify or dampen demand risk. If the firm adds the less responsive forward-buying channel via dual sourcing, which intensifies demand risk, the flexibility to control demand via pricing becomes more beneficial, so dynamic pricing and dual sourcing are strategic complements. Conversely, if the

more responsive spot-purchasing channel is added, dynamic pricing becomes less valuable, and thus the two strategies are substitutes.

7.2. Extensions

There are several ways to extend our model: (a) a model with the more general demand $D_t = d_t \epsilon_t^m + \epsilon_t^a$, (b) a model with long-term supply contract, and (c) a model with fixed ordering costs.

First, we consider a more general demand form: $D_t = d_t \epsilon_t^m + \epsilon_t^a$ where $\{\epsilon_t^m\}$ and $\{\epsilon_t^a\}$ are i.i.d. random variables with $\mathbb{E}[\epsilon_t^m] = 1$ and $\mathbb{E}[\epsilon_t^a] = 0$ (see, e.g., Chen and Simchi-Levi 2004a, Yang and Zhang 2014). The optimal policy structures characterized in Theorem 1 remain the same. Moreover, Theorem 3 continues to hold; i.e., if the spot market procurement cost evolves according to a concavely increasing stochastic function, a risk-neutral firm prefers a more volatile procurement cost process. However, the impact of current procurement costs, future procurement cost trends, and dual-sourcing flexibility upon the optimal pricing and inventory decisions of the firm can no longer be characterized.

Second, we can extend the model to one with long-term supply contract. If the firm signs a long-term supply contract that specifies the delivery quantity and cost in each period, all of our analytical and numerical results continue to hold. Adding the prespecified ordering quantity and payment only changes the feasible set of the decision variables but does not affect the structure of the objective and value functions. However, solving the optimal procurement quantity and cost in the long-term contract prior to the planning horizon is very difficult and involves a $2T$-dimensional nonlinear program.

Finally, we consider the extension to a model with fixed ordering costs. If the firm adopts the sole-sourcing strategy, which replenishes inventory only from the spot market with a fixed ordering cost, the model is an extension of Chen and Simchi-Levi (2004a). Following their approach, we can show that a cost-dependent (s, S, p) policy is optimal. We can also establish that when a fixed ordering cost applies, a risk-neutral firm may earn higher expected profit under a more volatile procurement cost process. However, other results regarding the impact of cost fluctuation on the firm's optimal policy may not hold. On the other hand, if the firm employs the dual-sourcing strategy, which procures inventory both from the spot market and through the forward-buying contract with fixed ordering costs, the problem becomes prohibitively difficult, and the optimal policy structure is unclear.

Supplemental Material

Supplemental material to this paper is available at http://dx .doi.org/10.1287/msom.2015.0519.

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